

تصحيح الفروض المحروس رقم 2

التمرين 1

$$f(x) = x + 2 - \sqrt{x^2 - 2x + 4}$$

$$D_f = \mathbb{R}$$

$$\forall x \in \mathbb{R} \quad x^2 - 2x + 4 > 0 \quad (*)$$

$$\lim_{x \rightarrow +\infty} f(x) = \lim_{x \rightarrow +\infty} \frac{(x+2) - \sqrt{x^2 - 2x + 4}}{(x+2) + \sqrt{x^2 - 2x + 4}}$$

$$= \lim_{x \rightarrow +\infty} \frac{6x}{(1 + \frac{2}{x} + \sqrt{1 - \frac{2}{x} + \frac{4}{x^2}}) x}$$

$$= \frac{6}{1} = 6$$

$$\lim_{x \rightarrow -\infty} f(x) = -\infty$$

$$\lim_{x \rightarrow -\infty} x + 2 = -\infty$$

$$\lim_{x \rightarrow -\infty} \sqrt{x^2 - 2x + 4} = +\infty$$

$$\lim_{x \rightarrow -\infty} \frac{f(x)}{x} = \lim_{x \rightarrow -\infty} \frac{x + 2 - \sqrt{x^2 - 2x + 4}}{x}$$

$$= \lim_{x \rightarrow -\infty} 1 + \frac{2}{x} + \frac{\sqrt{1 - \frac{2}{x} + \frac{4}{x^2}}}{1}$$

$$= 2$$

$$\lim_{x \rightarrow -\infty} f(x) - 2x = \lim_{x \rightarrow -\infty} (2 - x) - \sqrt{x^2 - 2x + 4}$$

$$= \lim_{x \rightarrow -\infty} \frac{-2x}{\frac{2}{x} - 1 - \sqrt{1 - \frac{2}{x} + \frac{4}{x^2}}}$$

$$= 1$$

$$\lim_{x \rightarrow -\infty} f(x) = 2x + 1$$

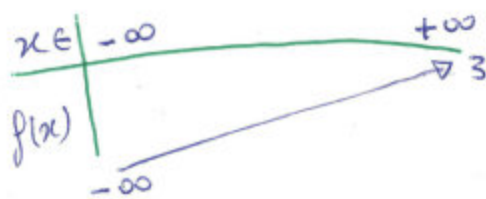
$$f'(x) = (x+2) - \frac{(x^2 - 2x + 4)}{2\sqrt{x^2 - 2x + 4}}$$

$$f'(x) = 1 - \frac{2x - 2}{2\sqrt{x^2 - 2x + 4}} = 1 - \frac{x - 1}{\sqrt{x^2 - 2x + 4}}$$

$$(x-1)^2 < x^2 - 2x + 4$$

$$x-1 < |x-1| < \sqrt{x^2 - 2x + 4}$$

$f'(x) > 0$ و f متزايدة في \mathbb{R}



$$f(x) - x = \frac{-x^2 + 2x}{2 + \sqrt{x^2 - 2x + 4}}$$

$\forall x \in \mathbb{R} -]0; 2[\quad -x^2 + 2x > 0$

$\forall x \in]0; 2[\quad -x^2 + 2x > 0$



$$U_{n+1} = f(U_n) \quad U_0 = 1$$

$$U_0 = 1$$

$$0 < U_n < 2 \quad \&$$

$$0 < U_{n+1} < 2$$

لدينا $0 < U_n < 2$ و f متزايدة

$$f(0) < f(U_n) < f(2)$$

$$0 < U_{n+1} < 2$$

$$0 < U_n < 2$$

$\forall x \in]0; 2[\quad f(x) > x$ لدينا

$$f(U_n) > U_n \quad U_n \in]0; 2[$$

$$U_{n+1} > U_n$$

تزايدية $(U_n)_n$

و $(U_n)_n$ متزايدة

$$U_0 = 1 \text{ متجاوزة } U_0$$

$$\forall n \in \mathbb{N} \quad 1 \leq U_n < 2$$

$$|U_{n+1} - 2| = |U_n - \sqrt{U_n^2 - 2U_n + 4}|$$

$$= \frac{2|U_n - 2|}{U_n + \sqrt{U_n^2 - 2U_n + 4}}$$

$$\sqrt{U_n^2 - 2U_n + 4} \geq \sqrt{3} \geq \frac{3}{2} \quad U_n \geq 1$$

$$\frac{1}{U_n + \sqrt{U_n^2 - 2U_n + 4}} < \frac{2}{5}$$

$$|U_{n+1} - 2| \leq \frac{4}{5} |U_n - 2|$$

و من اجل $n=0$

$$|U_{n-2} - 2| = |-1 - 2| = 1 < 1$$

$$|U_n - 2| \leq \left(\frac{4}{5}\right)^n$$

$$|U_{n+1} - 2| \leq \left(\frac{4}{5}\right)^{n+1}$$

$$|U_{n+1} - 2| \leq \frac{4}{5} |U_n - 2|$$

$$|U_n - 2| \leq \left(\frac{4}{5}\right)^n$$

$$|U_{n+1} - 2| \leq \left(\frac{4}{5}\right)^{n+1}$$

$$\forall x \in \mathbb{N} \quad |U_n - 2| \leq \left(\frac{4}{5}\right)^n$$

$$U_n < 2 \quad |U_n - 2| \leq \left(\frac{4}{5}\right)^n$$

$$\frac{4}{5} U_n < 2 \leq U_n + 2 \leq \left(\frac{4}{5}\right)^n$$

$$\sum_{k=0}^{n-1} 2 - \sum_{k=0}^{n-1} \left(\frac{4}{5}\right)^k \leq U_n \leq \sum_{k=0}^{n-1} 2$$

$$2n - 5 \left(\frac{4}{5}\right)^n \leq U_n \leq 2n$$

$$2 - \frac{5}{n} - \frac{4^n}{5^{n+1}} \leq U_n \leq 2 \quad \Leftarrow$$

التمرين 2

$$T_n = \sum_{k=0}^n \frac{(-1)^k}{2k+1}$$

$$U_0 = 1$$

$$U_0 = \frac{2}{3}$$

$$\begin{aligned} U_n - U_{n+1} &= T_{2n} - T_{2n+1} \\ &= \sum_{k=0}^{2n} \frac{(-1)^k}{2k+1} - \sum_{k=0}^{2n+1} \frac{(-1)^k}{2k+1} \\ &= \frac{(-1)^{2n+1}}{2n+3} > 0 \end{aligned}$$

$$\forall n \in \mathbb{N} \quad U_n < U_{n+1}$$

$$\begin{aligned} U_{n+1} - U_n &= T_{2n+1} - T_{2n} \\ &= \sum_{k=0}^{2n+1} \frac{(-1)^k}{2k+1} - \sum_{k=0}^{2n} \frac{(-1)^k}{2k+1} \\ &= \frac{1}{2n+3} - \frac{1}{2n+1} > 0 \end{aligned}$$

$$\begin{aligned} U_{n+1} - U_n &= T_{2n+1} - T_{2n} \\ &= \frac{(-1)^{2n+1}}{2n+3} + \frac{(-1)^{2n}}{2n+1} \\ &= \frac{1}{2n+3} - \frac{1}{2n+1} < 0 \end{aligned}$$

$$T_n = T_{2p} = U_p \quad n=2p$$

$V_n < U_n$ تناقصية (V_n)

$$V_0 = \frac{2}{3} \leq T_n = U_p \leq 1 = U_0 \quad n=2p+1$$

$$T_n = U_p$$

$$\frac{2}{3} = U_0 \leq V_p < U_p \leq U_0 = 1$$

$$\frac{2}{3} \leq T_n \leq 1$$

محدودة (T_n)

التمرين 3

$$1 = x \cdot y \Rightarrow x=1 \text{ و } y=1$$

$$h(1) = 2h(1)$$

$$h(1) = 0$$

$$1 = h'(1) = \lim_{x \rightarrow 1} \frac{h(x)}{x-1}$$

ليكن a عدداً حقيقياً موجباً وقابلية استيفاق n في 0

$$\lim_{x \rightarrow a} \frac{h(x) - a}{x - a} = \lim_{x \rightarrow a} \frac{h(x) - h(a)}{a \left(\frac{x-a}{a} \right)}$$

$$x \rightarrow 1 \quad x \rightarrow a \quad x = \frac{x}{a}$$

$$\lim_{x \rightarrow 1} \frac{1}{a} \frac{h(xa) - h(a)}{x-1}$$

$$\lim_{x \rightarrow a} \frac{1}{a} \left(\frac{h(x)}{x-1} \right) = \frac{1}{a}$$

n قابلية الاستيفاق على 0 موجباً
 $f'(x) = \frac{1}{x}$

التمرين 4

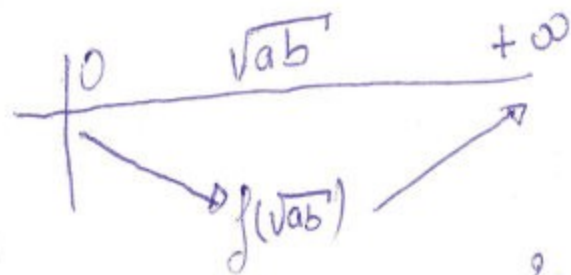
$$\forall x \in]0; +\infty[\quad \text{زنج}$$

$$f(x) = a^3 + b^3 + x^3 - 3abx$$

لدينا

$$f'(x) = 3x^2 - 3ab = 3(x^2 - ab)$$

أي



$$f(\sqrt{ab}) = (a\sqrt{a} - b\sqrt{b})^2 \geq 0$$

$$\forall x \in]0; +\infty[: f(x) \geq 0 \quad \text{أي}$$

$$c \in]0; +\infty[\quad \text{لدينا}$$

$$f(c) = a^3 + b^3 + c^3 - 3abc \geq 0$$

$$a^3 + b^3 + c^3 \geq 3abc \quad \text{و لهذا}$$